AP STAT- CHAPTER 4
PROBABILITY- the study of randomness!

Intro Vocab:

Random (trials)-

individual outcomes are uncertain, but there is a regular distribution in a large # of trials

Probability-

Proportion of times the outcome would occur in a large number of trials

Experimental Probability-

What DID happen in an expt. Also, the probability or percent of times an event occurs in an expt.

Ex: If I toss a coin 30 times, and get 12 heads, what the experimental prob. of getting heads? 12/30 = 2/5 = 0.4

Theoretical Probability-

What SHOULD happen. Also, the probability that should occur for an outcome in an expt.

Ex: Using the same coin tossing situation above, whats the theoretical prob. of getting heads? ½ = 0.5

Probability Models-

* A list of all possible outcomes

* The probability of each outcome

Sample Space-

The set of all possible outcomes in an experiment

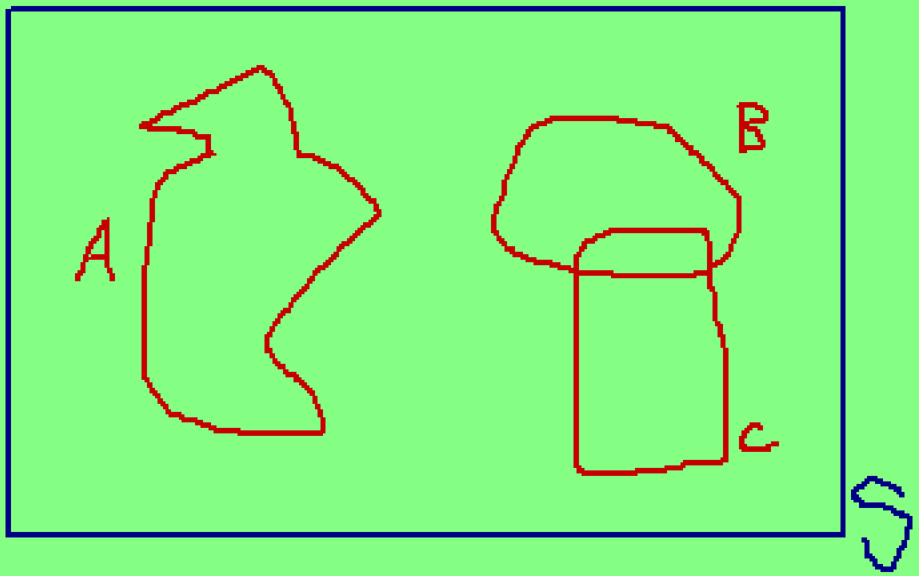
Examples: #1) If I am rolling a dice once, the sample space would be {1,2,3,4,5,6}

#2) If I am flipping a coin 3 times, it would be:

HHH
HHT
HTH
THH
TTH
THT
HTT
TTT

Probability Notation:

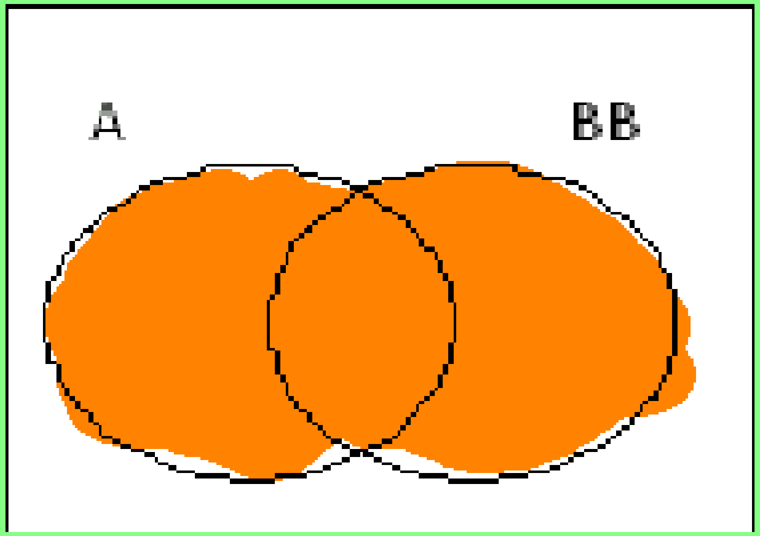
- A, B, C, etc. = events or outcomes
- P(A) = the probability of event A occuring
- S = the letter we use to represent the sample space
- When we represent events, we draw them with Venn Diagrams
- Venn Diagrams use shapes to represent events and a box around the shapes that represents the sample space
  - Examples:



General Set Theory

Union: **AND**

- Meaning: joining, addition, (marriage)
- Symbol: **U**
- Example 1:



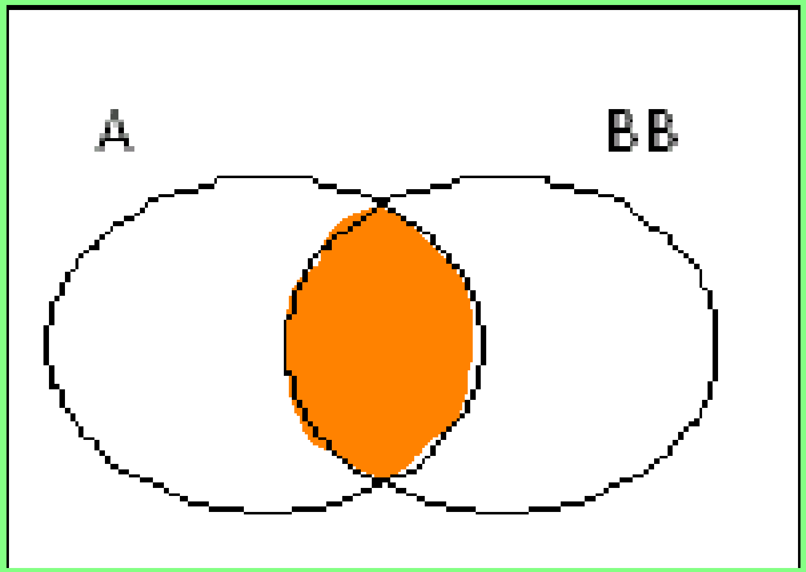
A U B = shaded

- Example 2: Set A = {2, 4, 6, 8, 10, 12}
  Set B = {1, 2, 3, 4, 5, 6, 7}

  A U B = { **1,2,3,4,5,6,7,8,10,12** }

Intersection: OR

- Meaning: overlap, things in common (like the intersection of 2 roads)
- Symbol: $\bigcap$
- Example 1:



A $\cap$ B = shaded

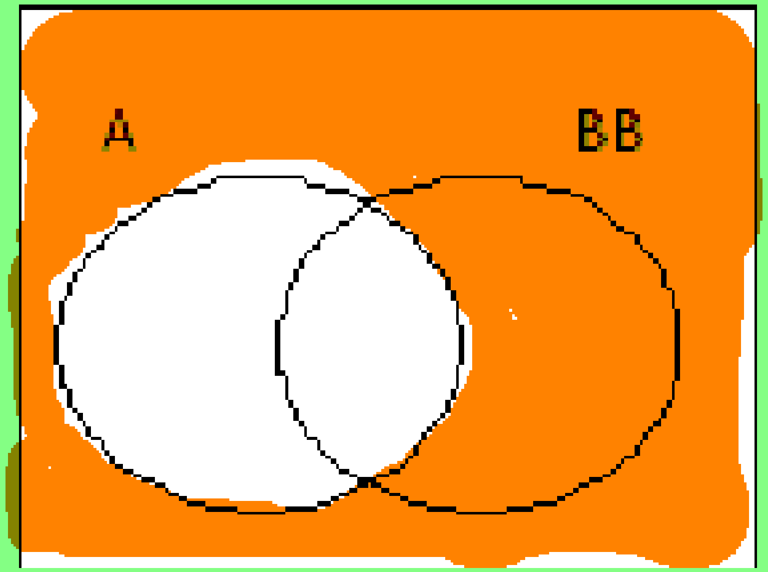
- Example 2: Set A = {2, 4, 6, 8, 10, 12}
  Set B = {1, 2, 3, 4, 5, 6, 7}

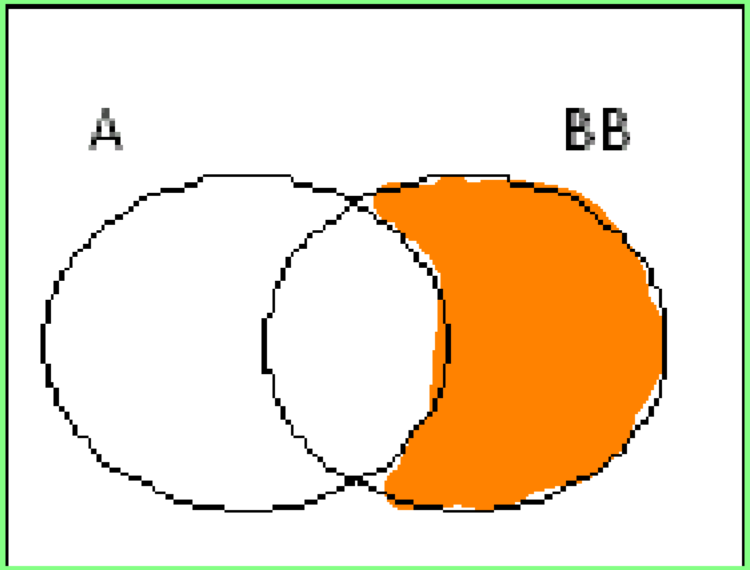
  A $\cap$ B = { 2, 4, 6 }

Complement: (of event A)

- Meaning: The complement of A is the probability of A NOT occuring. It is everything BUT event A.

- Symbol: A^c ←
  - Example 1: Shade A^c (everything except A) Shade $A^c \cap B$ (where "not A" intersects or overlaps with B)





- Example 2: Set A = {2, 4, 6, 8, 10, 12}

  S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} *=sample space*

  A^c = { 1, 3, 5, 7, 9, 11, 13, 14, 15 }

Try the SET THEORY worksheet.
Read all directions CAREFULLY!!

Probability Rules

- Let A and B be events
- Let S = sample space
- Let A^C = the complement of event A

List the first 3 probability rules: (page 298)

(1) $0 \leq P(A) \leq 1$

(2) $P(S) = 1$

(3) $P(A^C) = 1 - P(A)$

Example 1: If the probability of hitting a homerun is 30%, whats the probability of not hitting a homerun?

$P(H) =$ 0.30

$P(H^C) = 1 - P(H) = 1 - 0.3 = 0.7$

Example 2: If there are only 8 different blood types, fill in the chart below:

Type	A+	A-	B+	B-	AB+	AB-	O+	O-
Probability	0.16	0.14	0.19	0.17	?	0.07	0.1	0.11

Add up all blood types listed and subtract from 1 to get the missing one.

1 - 0.94 = 0.06 for blood type AB+

Example 3: Las Vegas Zeke, when asked to predict the ACC basketball Champion, follows the modern practice of giving probabilistic predictions. He says, "UNC's probability of winning is twice Duke's. NC State and UVA each have probability 0.1 of winning, but Duke's probability is three times that. Nobody else has a chance." Has Zeke given a legitimate assignment of probabilities to all the teams in the conference? Why or why not?

UNC = 0.6
Duke = 0.3
NCState = 0.1
UVA = 0.1

Add up all probabilities = 1.1

NO!! All probabilities must add up to 1 (100%). These add up to over 1.

Probability Rules (cont'd)

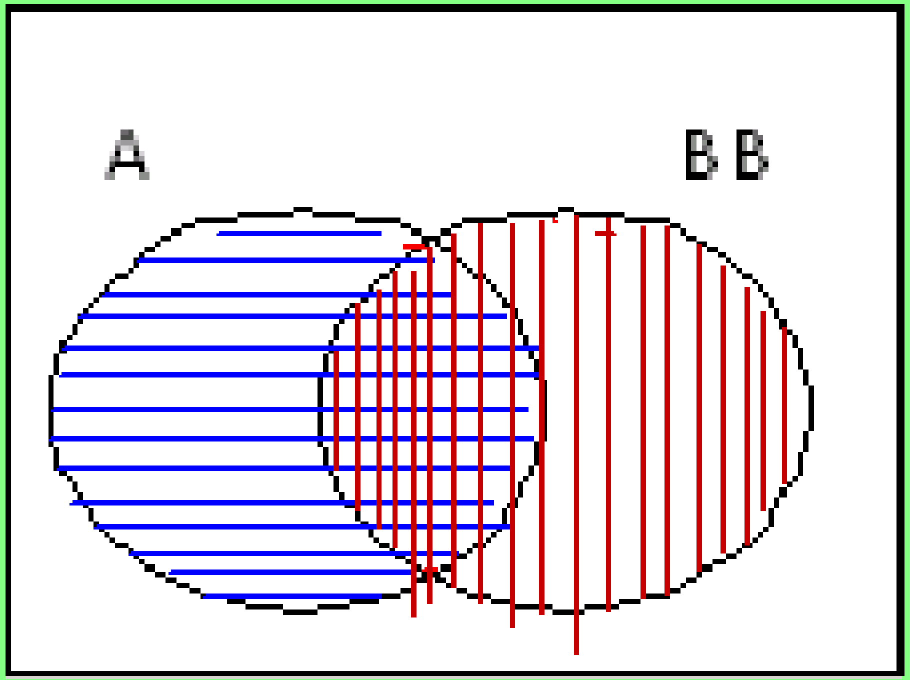
Unions Addition!

General Rule:

- $P(A \cup B) =$ $P(A) + P(B) - P(A \cap B)$

- Why do we subtract $P(A \cap B)$?

We don't want to double count the intersection (overlap of A and B)

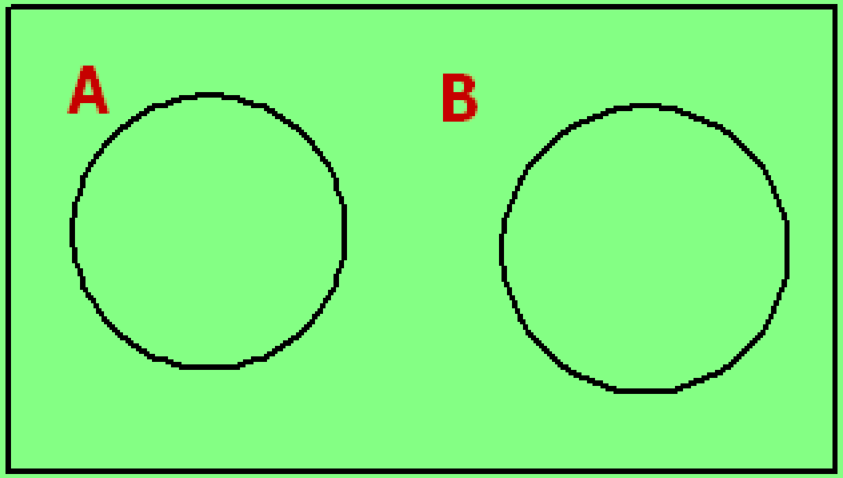


P(A) = blue
P(B) = red

**** notice that their intersection is counted twice when you shade (add) the P(A) to the P(B) to make the P(A U B)**

Special Case:

What if A and B don't overlap? Draw a Venn Diagram that illustrates this below:



So, $P(A \cap B)$ = **0**

This is called **<u>Disjoint</u>**

Disjoint (or mutually exclusive)= Two events are disjoint if ...

they have no outcomes in common

Example: rolling dice and flipping coin, or picking a jack and picking a 2

Not example: picking a jack and picking a black card

So our rule for unions for disjoint events then becomes:

- P(A U B) = P(A) + P(B)

Probability Rules (cont'd)

means "given that"

Conditional Probability = P(B|A)= Probability of B happening given that A hapened

Definition:

Giving the probability of an event with the knowledge that another event already happened. Sometimes, a first event happening will affect the chance of a second event happening.

Formula:

$$P(B|A)= \frac{P(A \cap B)}{P(A)}$$

Notes:

- $P(A) > 0$
- A = first event
- B = second event

Example: Picking poker chips from a bag:
I have a bag that has 10 poker chips in it. 3 are red, 2 are green, and 5 are blue. I am picking 2 poker chips out. What's the probability of picking a blue chip then a red chip? (**WITH replacement** in between picks) (whats the chance you pick a red second, knowing you picked a blue first and then replaced it)

P(A) = (prob of picking a blue chip) = 5/10

P(B) = (prob of picking a red chip) = 3/10

P(B|A) = (prob of picking a red chip given that you picked a blue chip already) = 3/10

* doesnt change cause you put the blue chip back!)

Now I change my situation so **I DO NOT REPLACE** the chips in between picks. *(this is conditional probability- the first event happening affects the chance of the second event happening)*

P(B|A) = (prob of picking a red chip given that you picked a blue chip already) = 3/9

* if the blue chip is picked and not replaced, there are only 9 chips left. 3 are still red.

** notice these are different probabilities!

Question: P(B|A) = P(A|B) ?? NO!!

Check this with your example above if needed! 3/9 does not equal 5/9

Intersections

General Rule:

- $P(A \cap B) =$ $P(A) * P(B|A)$

Rearragnge formula from conditional probabilities by multiplying both sides of that formula by P(A). Then you will have this formula!

- Note: A = 1st event

  B = 2nd event

- Also called... multiplication rule

Special Case:
What if A and B don't affect each other?

Using our example from above, what lets go back to where we **replaced** the chips after each pick, and here I am only focusing on picking one chip at a time.

What would be the probability of picking a red chip on the first pick? P(Red) = 3/10

If we picked a blue chip, on the first pick (but replaced it after the pick), what is the probability that we picked a red chip on the second pick? P(Red|Blue) = 3/10

How do these two probabilities (picking red on first and second picks) compare?

they are the same!

So in general, P(B|A) = P(B) (the first event A, doesn't matter!)

This is called **Independent**

Independent= Two events are independent if …

they don't affect each other! the 1st event happening doesn't affect the chance of the second event happening.

Examples of independent events:

Rolling dice and picking cards, or rolling the dice twice

So our rule for intersections for independent events then becomes:

- $P(A \cap B) =$ $P(A) * P(B)$

Example: tossing coin twice

$P(HH) = P(H) * P(H) = \frac{1}{2} * \frac{1}{2} = 1/4$

Do the "Try these:Probability Basics" sheet.
Use your notes and/or use your formula sheets.
the formulas are on the back of the first page of the formula sheet.

Formulas:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

** Special case: if A and B are disjoint, $P(A \cup B) = P(A) + P(B)$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$P(A \cap B) = P(A) * P(B|A)$

** Special Case: if A and B are independent, $P(A \cap B)= P(A)*P(B)$